AP Calculus Summer Assignment Name ___________________________

Pre-Calculus Review

As a review, there are 20 Pre-Calculus topics that you need to know and master before you start preparing for AP Calculus. This packet is not meant to be a comprehensive review and if any of these topics are confusing, please ask Mr. Kathman for support.

1) FUNCTIONS

The life blood of Pre-Calculus and Calculus is functions. A function is defined as the set of points (x, y) such that for every x, there is one and only one y. In a function, the $x\ values$ cannot repeat and the $y\ values$ can repeat.

a) If $f(x) = x^2 - 5$ and $g(x) = 2x + 1$, show that $f(g(x)) \neq g(f(x))$

2) DOMAIN and RANGE

In Calculus, there is much interest in the behavior of functions on specific intervals. Understand that intervals can be written with a description in terms of $<, >, \leq, \geq$, or by using interval notation.

$Description$	$Interval\ Notation$
$x > a$	(a, ∞)
$x \geq a$	$[a, \infty)$
$x < a$	$(-\infty, a)$
$x \leq a$	$(-\infty, a]$
$a < x < b$	$(a, b) - open\ interval$
$a \leq x \leq b$	$[a, b] - closed\ interval$
$a \leq x < b$	$[a, b)$
$a < x \leq b$	$(a, b]$
$all\ reals$	$(-\infty, \infty)$

a) If $f(x) = \frac{x^2+4x+6}{\sqrt{2x+4}}$, use both inequalities and interval notation to describe the domain and range.

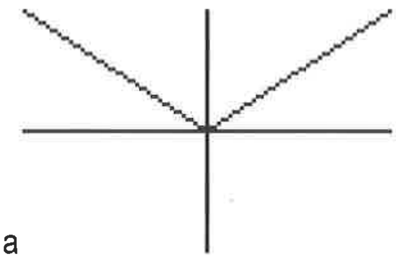
b) Find the domain and range for each of the functions in the FUNCTION TOOLBOX section. Use inequalities and interval notation. Write your answers next to the graphs on the following pages.

AP Calculus Summer Assignment Name ____________________

Pre-Calculus Review

3) FUNCTION TOOLBOX

Certain graphs appear regularly in Calculus and students should know the general shape, where they intersect the $x - axis\ (roots)$ and $y - axis\ (y - intercept)$.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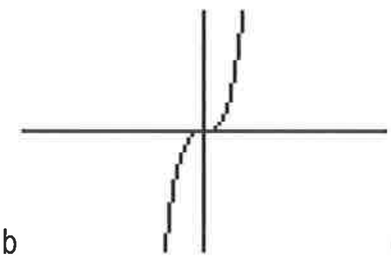
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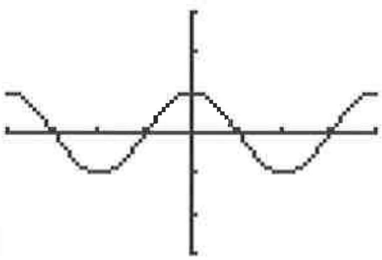
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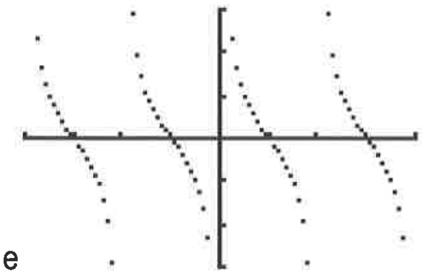
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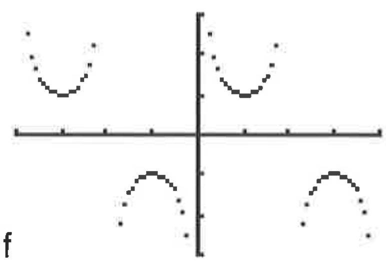
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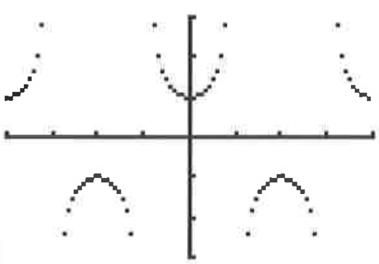
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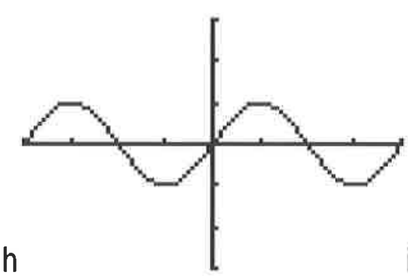
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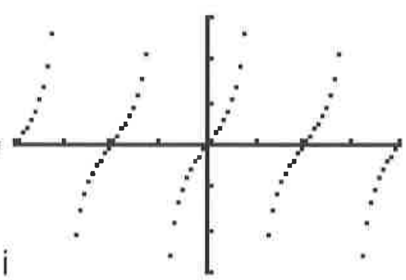
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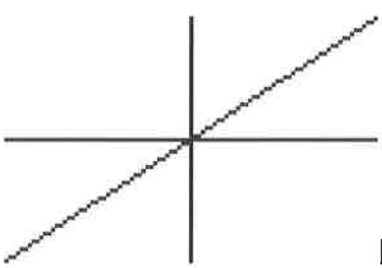
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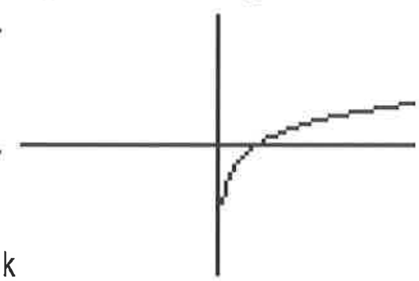
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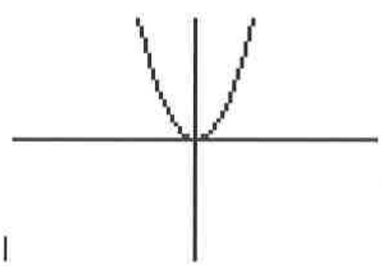
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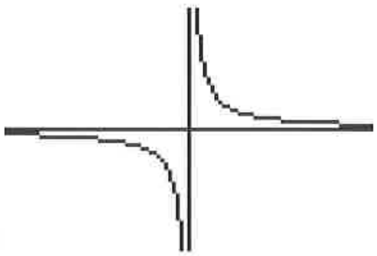

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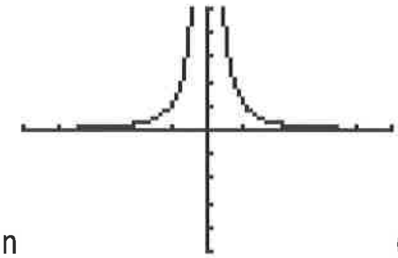

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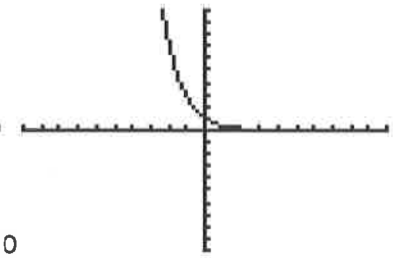

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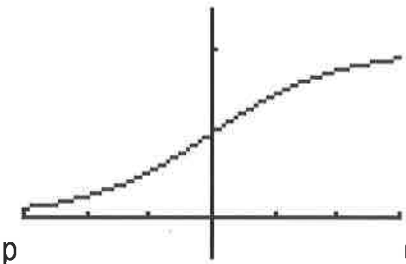

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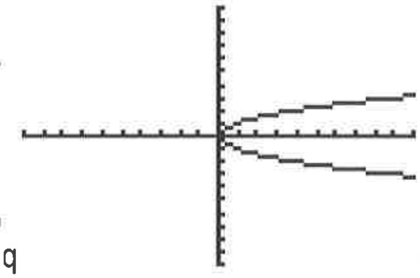

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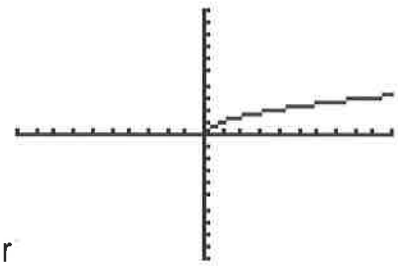

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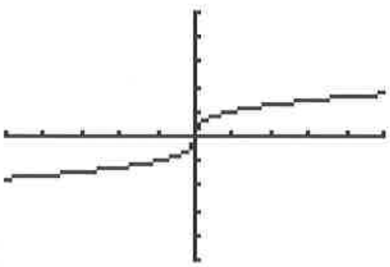

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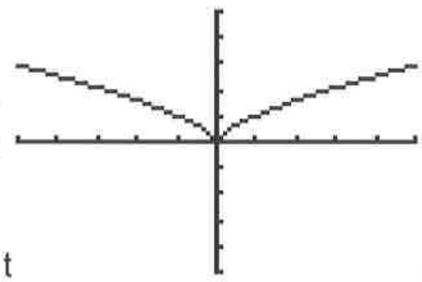

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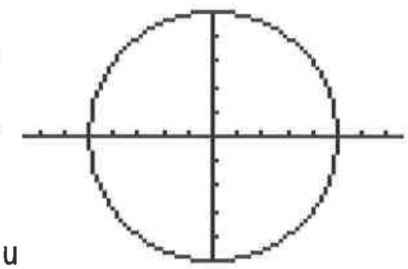

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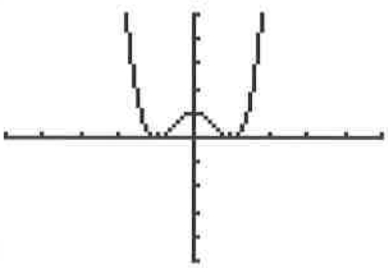

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Match each function definition with its graph. State the domain, range, and any intercepts for each function.

i. _____ $f(x) = e^{-x}$

ii. _____ $f(x) = e^{x}$

iii. _____ $f(x) = x$

iv. _____ $f(x) = x^2$

v. _____ $f(x) = x^3$

vi. _____ $f(x) = x^4$ $[f(x) = (x+1)^2(x-1)^2]$

vii. ______ $f(x) = \sqrt{x}$

viii. ______ $f(x) = \sqrt[3]{x}$

ix. ______ $f(x) = \sqrt[3]{x^2}$

x. ______ $f(x) = sin(x)$

xi. ______ $f(x) = cos(x)$

xii. ______ $f(x) = tan(x)$

xiii. ______ $f(x) = csc(x)$

xiv. ______ $f(x) = sec(x)$

xv. ______ $f(x) = cot(x)$

xvi. ______ $f(x) = ln(x)$

xvii. ______ $f(x) = \frac{1}{x}$

xviii. ______ $f(x) = \frac{1}{x^2}$

xix. ______ $f(x) = \frac{1}{1+e^{-x}}$

xx. ______ $f(x) = |x|$

xxi. ______ $x^2 + y^2 = 25$

xxii. ______ $x = y^2$

4) EVEN and ODD FUNCTIONS

Functions that are even have the characteristic that for all a, $f(-a) = f(a)$. Even functions are symmetric to the $y - axis$.

Functions that are odd have the characteristic that for all a, $f(-a) = -f(a)$. Odd functions are symmetric to the origin.

a) Of the functions in section 3, which are even, which are odd and which are neither?

5) TRANSFORMATIONS

A curve in the form $y = f(x)$ can be transformed in a variety of ways. The shape of the resulting curve stays the same, but roots and intercepts may change. Additionally, the resulting curve may be reflected about an axis.

6) SPECIAL FACTORIZATION

Difference of Squares:	$a^2 - b^2 = (a-b)(a+b)$
Perfect Squares:	$a^2 + 2ab + b^2 = (a+b)^2$
	$a^2 - 2ab + b^2 = (a-b)^2$
Sum of Cubes:	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
Difference of Cubes:	$a^3 - b^3 = (a-b)(a^2 + ab + b^2$

7) LINEAR EQUATIONS

One of the most important concepts required for AP Calculus is linear equations. Important formulas are listed.

Slope Formula $m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Slope-intercept Form $y = mx + b$

Point-slope Form $y - y_1 = m(x - x_1)$

Two distinct lines are parallel if they have the same slope and two lines are normal (perpendicular) if their slopes are opposite reciprocals.

8) SOLVING QUADRATIC EQUATIONS

Solving quadratics of the form $ax^2 + bx + c = 0$ usually shows up on the AP exam in the form of expressions that can be factored easily. If the quadratic does not factor or factoring is too time consuming, use the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a \neq 0$$

9) ASYMPTOTES

Rational functions of the form $y = \frac{p(x)}{q(x)}$ may have vertical asymptotes. Vertical asymptotes can be found by setting the denominator equal to zero and solving.
Horizontal asymptotes are lines the graph of the function approaches when x gets very large or very small.

10) INVERSES

If a function is a rule that maps x to y, an inverse is a rule that brings y back to the original x. If a point (x, y) is a point on the function f, then the point (y, x) is on the inverse functionf^{-1}. It is possible that the inverse of a function is not a function.

11) NEGATIVE AND FRACTIONAL EXPONENTS

In Calculus, you will be required to perform algebraic manipulations with negative exponents as well as fractional exponents. You should know $x^{-n} = \frac{1}{x^n}, where\ x \neq 0.$ Typically, expressions in multiple choice answers will be written with positive exponents and students are required to eliminate the negative exponents.

You should also know $x^{\frac{a}{b}} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$

12) COMPLEX FRACTIONS

Calculus frequently uses complex fractions, which are fractions within fractions. Answers are never left with complex fractions and they must be eliminated. While there are other methods, the best way to accomplish this is to find the LCD (least common denominator) of all the fractions in the complex fraction. Multiply all terms by this LCD and you are left with a fraction that is no longer complex.

13) SOLVING RATIONAL EQUATIONS

Multiply the entire equation by the LCD. Factors in the denominators of the fractions should cancel and you will be left with an equation (hopefully) that is easy to solve.

a) Solve $\frac{12}{x+2} - \frac{4}{x} = 1$

14) SOLVING ABSOLUTE VALUE EQUATIONS

To solve an absolute value equation, split the absolute value into two equations. One equation should be the same as the original with the absolute value removed and the other equation should be re-written with the quantity inside absolute value multiplied by a factor of -1.

15) SOLVING INEQUALTITIES

Solving inequalities is not just a matter of replacing the equal sign with an inequality symbol. In reality, they can be more difficult and are fraught with dangers.

Solving inequalities is a simple matter if the inequalities are based on linear equations. They are solved in the same manner as linear equations, remembering that if you multiply or divide by a negative, the inequality symbol must be reversed.

If the inequality is more complex than a linear function, it is advised to bring all of the terms to one side of the equation and graph the related function. Keep in mind that the inequality symbols $<, >$ yield a dotted line and the symbols $\leq, \geq$ yield a solid line. Shade the area of the graph which includes the solution set.

16) EXPONENTIAL AND LOGARITHMIC FUNCTIONS

If $y = b^x$, then $x = \log_b y$ OR if $y = e^x$, then $x = \log_e y = \ln y$.
You should remember $e = 2.71828$ ...

Properties of Logarithms, which are interchangeable with natural logs, you should know:

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log a^b = b \log a$$

a) Find $\log_4 8$

b) Find $\ln \sqrt{e}$

c) Find $\log 250 - \log 2.5$

d) Solve $\log_9(x^2 - x + 3) = \frac{1}{2}$

e) Solve $5^x = 20$

17) RIGHT TRIANGLE TRIGONOMETRY

Trigonometry is an important part of Calculus. You should know all six trigonometric functions and their ratios.

$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{hyp}{opp}$$

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{hyp}{adj}$$

$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

18) SPECIAL ANGLES

You should memorize the trigonometric function values of the special angles $(0°, 30°, 45°, 60°, 90°, 180°, 270°, and\ 360°\ OR\ 0\pi, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, and\ 2\pi)$. All can be found on your unit circle.

The relationship of side lengths in the special triangles $(30° - 60° - 90°\ and\ 45° - 45° - 90°)$ should also be memorized.

19) TRIGONOMETRIC IDENTITIES

You are not asked these identities directly on the AP exam, but you should have all identities listed below memorized. These are the identities that are most likely to appear on the AP exam.

$\tan x = \frac{\sin x}{\cos x}$ $\qquad$ $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cot x = \frac{\cos x}{\sin x}$ $\qquad$ $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $\sin(2x) = 2 \sin x \cos x$

$1 + \tan^2 x = \sec^2 x$ $\qquad$ $1 + \cot^2 x = \sec^2 x$

$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

20) TRIGONOMETRIC EQUATIONS

Typically these equations have infinitely many solutions, so usually a specific domain is provided. All answers should be written in $Radian$ measure.

a) Solve on the interval $[0, 2\pi)$

$$x \cos x = 2 \cos x$$

b) Solve on the interval $[0, 2\pi)$

$$\tan x + \sin^2 x = 2 - \cos^2 x$$